

The topic of chaos has become quite popular over the last few decades. A system is defined to be *chaotic* when its behavior is so sensitive to some factor that it is extremely difficult to predict. The field of *chaos* is the study of chaotic behavior. A good example of chaotic behavior is the flow of a fluid with a Reynolds number between 2000 and 3000. Whether or not the flow is turbulent is difficult, but not impossible, to predict—the difficulty lies in the extremely sensitive dependence on factors like roughness and obstructions on the nature of the flow. A tiny variation in one factor has an exaggerated (or nonlinear) effect on the flow. Phenomena as disparate as turbulence, the orbit of Pluto, and the onset of irregular heartbeats are chaotic and can be analyzed with similar techniques.

## 12.6 Motion of an Object in a Viscous Fluid

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number  $N'_R$ , defined for an object moving in a fluid to be

$$N'_R = \frac{\rho v L}{\eta} (\text{object in fluid}), \quad 12.55$$

where  $L$  is a characteristic length of the object (a sphere's diameter, for example),  $\rho$  the fluid density,  $\eta$  its viscosity, and  $v$  the object's speed in the fluid. If  $N'_R$  is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for  $N'_R$  between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an  $N'_R$  between 10 and  $10^6$ , the flow may be either laminar or turbulent and may oscillate between the two. For  $N'_R$  greater than about  $10^6$ , the flow is entirely turbulent, even at the surface of the object. (See [Figure 12.18](#).) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.



### EXAMPLE 12.10

#### Does a Ball Have a Turbulent Wake?

Calculate the Reynolds number  $N'_R$  for a ball with a 7.40-cm diameter thrown at 40.0 m/s.

#### Strategy

We can use  $N'_R = \frac{\rho v L}{\eta}$  to calculate  $N'_R$ , since all values in it are either given or can be found in tables of density and viscosity.

#### Solution

Substituting values into the equation for  $N'_R$  yields

$$\begin{aligned} N'_R &= \frac{\rho v L}{\eta} = \frac{(1.29 \text{ kg/m}^3)(40.0 \text{ m/s})(0.0740 \text{ m})}{1.81 \times 10^{-5} \text{ Pa}\cdot\text{s}} \\ &= 2.11 \times 10^5. \end{aligned} \quad 12.56$$

#### Discussion

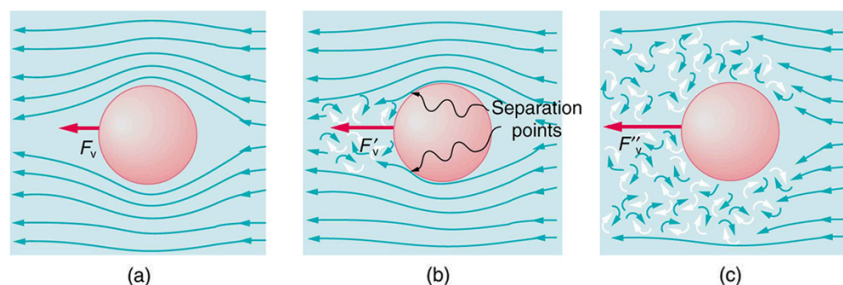
This value is sufficiently high to imply a turbulent wake. Most large objects, such as airplanes and sailboats, create significant turbulence as they move. As noted before, the Bernoulli principle gives only qualitatively-correct results in such situations.

One of the consequences of viscosity is a resistance force called **viscous drag**  $F_V$  that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow ( $N'_R$  less than about one) viscous drag is proportional to speed, whereas for  $N'_R$  between about 10 and  $10^6$ , viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For  $N'_R$  greater than  $10^6$ , drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere,  $F_V$  is proportional to fluid viscosity  $\eta$ , the object's characteristic size  $L$ , and its speed  $v$ . All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law  $F_S = 6\pi\eta r v$ . For the special case of a small

sphere of radius  $R$  moving slowly in a fluid of viscosity  $\eta$ , the drag force  $F_S$  is given by

$$F_S = 6\pi R\eta v.$$

12.57



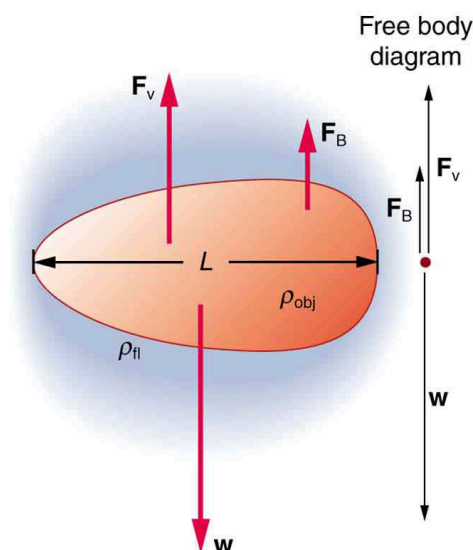
**Figure 12.18** (a) Motion of this sphere to the right is equivalent to fluid flow to the left. Here the flow is laminar with  $N'_R$  less than 1. There is a force, called viscous drag  $F_v$ , to the left on the ball due to the fluid's viscosity. (b) At a higher speed, the flow becomes partially turbulent, creating a wake starting where the flow lines separate from the surface. Pressure in the wake is less than in front of the sphere, because fluid speed is less, creating a net force to the left  $F'_v$  that is significantly greater than for laminar flow. Here  $N'_R$  is greater than 10. (c) At much higher speeds, where  $N'_R$  is greater than  $10^6$ , flow becomes turbulent everywhere on the surface and behind the sphere. Drag increases dramatically.

An interesting consequence of the increase in  $F_v$  with speed is that an object falling through a fluid will not continue to accelerate indefinitely (as it would if we neglect air resistance, for example). Instead, viscous drag increases, slowing acceleration, until a critical speed, called the **terminal speed**, is reached and the acceleration of the object becomes zero. Once this happens, the object continues to fall at constant speed (the terminal speed). This is the case for particles of sand falling in the ocean, cells falling in a centrifuge, and sky divers falling through the air. [Figure 12.19](#) shows some of the factors that affect terminal speed. There is a viscous drag on the object that depends on the viscosity of the fluid and the size of the object. But there is also a buoyant force that depends on the density of the object relative to the fluid. Terminal speed will be greatest for low-viscosity fluids and objects with high densities and small sizes. Thus a skydiver falls more slowly with outspread limbs than when they are in a pike position—head first with hands at their side and legs together.

### Take-Home Experiment: Don't Lose Your Marbles

By measuring the terminal speed of a slowly moving sphere in a viscous fluid, one can find the viscosity of that fluid (at that temperature). It can be difficult to find small ball bearings around the house, but a small marble will do. Gather two or three fluids (syrup, motor oil, honey, olive oil, etc.) and a thick, tall clear glass or vase. Drop the marble into the center of the fluid and time its fall (after letting it drop a little to reach its terminal speed). Compare your values for the terminal speed and see if they are inversely proportional to the viscosities as listed in [Table 12.1](#). Does it make a difference if the marble is dropped near the side of the glass?

Knowledge of terminal speed is useful for estimating sedimentation rates of small particles. We know from watching mud settle out of dirty water that sedimentation is usually a slow process. Centrifuges are used to speed sedimentation by creating accelerated frames in which gravitational acceleration is replaced by centripetal acceleration, which can be much greater, increasing the terminal speed.



**Figure 12.19** There are three forces acting on an object falling through a viscous fluid: its weight  $w$ , the viscous drag  $F_v$ , and the buoyant force  $F_B$ .

## 12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

### Diffusion

There is something fishy about the ice cube from your freezer—how did it pick up those food odors? How does soaking a sprained ankle in Epsom salt reduce swelling? The answer to these questions are related to atomic and molecular transport phenomena—another mode of fluid motion. Atoms and molecules are in constant motion at any temperature. In fluids they move about randomly even in the absence of macroscopic flow. This motion is called a random walk and is illustrated in [Figure 12.20](#). **Diffusion** is the movement of substances due to random thermal molecular motion. Fluids, like fish fumes or odors entering ice cubes, can even diffuse through solids.

Diffusion is a slow process over macroscopic distances. The densities of common materials are great enough that molecules cannot travel very far before having a collision that can scatter them in any direction, including straight backward. It can be shown that the average distance  $x_{\text{rms}}$  that a molecule travels is proportional to the square root of time:

$$x_{\text{rms}} = \sqrt{2Dt},$$

12.58

where  $x_{\text{rms}}$  stands for the **root-mean-square distance** and is the statistical average for the process. The quantity  $D$  is the diffusion constant for the particular molecule in a specific medium. [Table 12.2](#) lists representative values of  $D$  for various substances, in units of  $\text{m}^2/\text{s}$ .